Dynamics of Strings and Rods with Applications to Flagellar Motion

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Many bacteria and spermatozoa propel themselves by spinning or beating their flagella. Interesting bifurcations, such as planar waveform, helical waveform and planar and helical transition, have been observed in their motions [1-3]. Their motion can be understood in terms of the theory of strings and slender rods that have been studied for a long time; the nonlinear governing equations exhibit a rich array of solutions. According to linear theory, a heavy string can exhibit steady-state whirl only at its natural frequencies which form a discrete spectrum. The nonlinear theory, however, suggests that a string can undergo steady whirl at any frequency larger than the fundamental frequency and further that for each frequency between the n^{th} and the $(n+1)^{th}$ eigenvalue, there exist n distinct whirling modes.

In this presentation, we study the whirling [4] and whipping motions of a string with through quantitative experimental measurements that reveal the existence of steady states, trapped states, and chaotic motion. It is shown that steady motion exists only when the string whirls at its natural frequencies and that whirling motions for other frequencies exhibit rich dynamics that needs further exploration. Under planar excitation, bifurcations to whirling motion are demonstrated. The results are used to examine the underlying instabilities and the similarity between the motion that is dominated by gravitational-inertial loading on the string on the one hand and the viscous resistance dominated motion in the flagella on the other hand.

Reference

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