A MODEL FOR GROWING, VISCOELASTIC TISSUES WITH AN APPLICATION TO TUMOR GROWTH

Magdalena A. Stolarska

School of Mathematics
University of Minnesota
Minneapolis, MN 55455
stolarsk@math.umn.edu

Growth of biological tissues is a process that involves the interaction of various chemical species. Furthermore, recent experiments indicate that stress feedback from the mechanics to the reaction kinetics also influences growth. Understanding how the reaction kinetics and mechanics are correlated is essential to understanding a variety of processes, such as embryonic development, bone regeneration, and cancer growth. In this talk, I describe a general framework for modeling the interaction of reaction kinetics with the mechanics of a growing, viscoelastic tissue. The model of tissue mechanics allows for large deformations while the reaction kinetics are modeled by reaction-diffusion equations where the nature of the reactions is determined experimentally. Growth and material response are modeled by a multiplicative decomposition of the deformation gradient. The resulting coupled system of non-linear partial differential equations is solved in three dimensions using an updated Lagrangian finite element method.

The model and computational framework is applied to the growth of a tumor in an agarose gel. Two simulations related to the experiment of Helmlinger et al. [1] are presented. The first part of the experiment illustrates that a tumor spheroid grown in an agarose gel under isotropic conditions reaches an equilibrium size that depends on the stiffness of the gel. In the second part of the experiment, the tumor is grown in a capillary tube that is one centimeter in length and one millimeter in diameter. In this case the spherical symmetry of the tumor is broken, and the tumor grows into an ellipsoidal shape, with the primary growth in the direction of the capillary axis. The simulations of these experiment illustrate the distribution and effects of residual stresses from tumor growth in addition to verifying the proposed model.

References

[1] G. Helmlinger, P.A. Netti, H.C. Lichtenbeld, R.J. Melder, and R.K. Jain, "Solid stress inhibits the growth of multicellular tumor spheroids," *Nature Biotechnology.* **15**, 778–783, 1997.