A DIRECTOR-FIELD MODEL OF MEMBRANE PACKING IN MITOCHONDRIA

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In eukaryotic cells, oxydative phosphorylation - the synthesis of ATP (adenosine triphosphate) from ADP (adenosine diphosphate) and inorganic phosphate, occurs in the organelles called the mitochondria. The mitochondria are bound by two distinct membranes, the outer membrane and the inner membrane. The inner membrane carries the enzyme complex called ATP synthase and is the site of ATP synthesis. Until recently, the structure of the inner membrane was described by the "baffle model." In this model, the inner membrane protrudes into the mitochondrion interior in a "baffle-like" manner, folding into parallel leaves called cristae, similar to the bellows of an accordion. In contrast, results from 3D imaging technology, cryo-electron tomography, show that the cristae are not baffles with wide openings into the intermembrane space, but are extensively tubular in nature [1]. The cristae have narrow, usually multiple tubular openings (crista junctions) to the intermembrane space, which led to this model being called the "crista junction model." The controversy over the organization of the membranes within the mitochondria has important implications on the complete understanding of the mitochondrial energy transduction process.

In this paper we present a numerical study of the mechanics of the inner membrane organization in the mitochondria. We formulate a continuum theory of the membrane packing in the mitochondria in terms of a director field representation of the inner membrane. The conformation of the inner membrane of the mitochondria is described completely by the director field, whose values are the direction of the membrane normal and the density of membrane folds at every point within the organelle. Assuming that the area of the membrane is conserved during deformation, the bending energy of the membrane can be defined as a functional of the director field. The director field that minimizes the energy of the inner membrane may then be found through numerical optimization. This approach has previously been applied successfully to the problem of DNA packaging in viral capsids [2], [3].

We consider two representative shapes of mitochondria, spherical and cylindrical, and search for the director field that minimizes the bending energy of the membrane. The energy functional is discretized on a finite-difference grid, and the approximate minimum energy configurations of the discretized problem are found through the simulated annealing and conjugate gradient methods.

References

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