MASS TRANSPORT IN WATER WAVES OVER A THIN LAYER OF VISCOELASTIC MUD

XUEYAN ZHANG* AND CHIU-ON NG †

* Department of Mechanical Engineering
The University of Hong Kong
Pokfulam, Hong Kong

† Department of Mechanical Engineering
The University of Hong Kong
Pokfulam, Hong Kong
cong@hku.hk

Cohesive bed-mud in coastal waters tends to exhibit vastly different rheological behaviors depending on the site conditions. It is convenient, as far as modeling is concerned, if the mud can be generalized into a material possessing both viscous and elastic characteristics. In this connection, the viscoelastic Voigt model has been commonly adopted in studies on the wave–mud interaction.

Although a viscoelastic bed is increasingly recognized as an important type of marine bed, and has been extensively studied for its first-order effects on the waves, its effects on the second-order mass transport in waves are by contrast much less studied. Some researchers have attempted to work out the transport rate of viscoelastic mud under wave action, but they have all mistakenly applied the complex viscoelastic parameter to the second-order problem. As a result, there virtually exists little work in the literature that has specifically examined mass transport in water waves over a viscoelastic bed. This forms the motivation and objective of the present study.

This paper aims to present a theory for the mass transport or Lagrangian drifts induced by a small-amplitude progressive wave propagating in water over a thin layer of viscoelastic mud modeled as a Voigt medium. Based on a sharp contrast in length-scales near the bed, boundary-layer equations of motion in Lagrangian form are solved for the first-order oscillatory motions in the mud and the near-bed water layers. On extending the analysis to the second order for the mass transport, it is pointed out that it is inappropriate as in some previous studies to apply the complex viscoelastic parameter to a higher-order analysis, and also to suppose that a Voigt body can undergo continuous steady motion. In fact, the time-mean motion of a Voigt body is only transient, and will stop after a time scale given by the ratio of the viscosity to the shear modulus. Once the mud has attained its steady deformation, the mass transport in the overlying water column can be found as if it was a single-layer system.

It is found that mass transport in water waves over a thin layer of viscoelastic material is qualitatively different from the cases when the material is purely viscous or purely elastic with infinite depth. When the material is purely viscous, the mass transport in the water layer decreases mildly, and remains positive. In sharp contrast, a viscoelastic material can lead to a significant change in the magnitude of the mass transport in the water layer, and even a change in the drift direction. It is remarkable that mass transport over a finite elastic layer can be dramatically different from that over an infinitely deep elastic bottom, since resonance can occur in the former, but not in the latter. Also, an elastic bed of infinite depth is always to increase the mass transport in the water layer, while an elastic bed of finite depth may increase or decrease the mass transport in the water layer depending on, among others, whether or not the depth is smaller than the first resonant depth. It is found that resonance, when occurring in the bottom layer, can play a dominant role in determining the magnitude as well as direction of the mass transport in the water column.

Keywords: wave–mud interaction, mass transport in water waves, viscoelastic mud