EXACT SOLUTION TO NOZZLE FLOWS OF DENSE GASES

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In this paper, the flow of dense gases in convergent-divergent nozzles is studied. In particular, we are interested in investigating the real-gas effects in flows of molecularly complex fluids operating near the liquid-vapor saturation curve. As it is well known, for molecularly complex fluids, such as for example Toluene (C_7H_8), non standard gasdynamic behaviour may be observed in these regimes, including the nonmonotone Mach distribution along the divergent section of the nozzle. For very complex fluids, such as PP10 ($C_{13}F_{22}$, pf-perhydroflourene), nonclassical phenomena may possibly occur, including rarefaction shock waves and sonic shocks in both the convergent and divergent sections.

A novel solution technique for nozzle flows of real gases is presented and applied to the computation of the flow of a van der Waals gas in a convergent-divergent nozzle under the quasi-onedimensional approximation. Following previous studies [1,2], the solution procedure is based on the mass velocity $j(\rho) = \rho u(\rho)$, with ρ density and u velocity. However, no approximation of $j(\rho)$ around sonic states are introduced here. In isentropic regions, the flow variables are evaluated from isentropic flow relation obtained by analytical integration of the Euler equations for compressible fluids. The integration constants are the total enthalpy, which is computed from reservoir conditions, the mass flow rate, which depends on the reservoir conditions and the ambient/exit pressure, and the (constant) entropy. A standard bisection method is used to compute the solution.

If shock occurs, the match between two (or more) isentropic regions connected by a shock wave is found by imposing suitable jump equations for the boundary values of the unknowns, namely, behind and past the shock, and for the (unknown) position of the shock itself. The resulting nonlinear system of algebraic equations is solved by means of a Newton iterative scheme. In this case, the integration constant associated with the isentropic region past the shock are computed as follows: the total enthalpy from reservoir conditions, the mass flow rate from reservoir conditions and the ambient/exit pressure and the entropy is obtained from post-shock conditions.

Nozzle flows of van der Waals gases are computed in both the dilute and dense gas limit, for different fluids of increasing molecular complexity. Different nonclassical solutions are presented and agree with the classification scheme proposed in [3]. Thanks to the algebraic approach followed here, the dependence on the ambient/exit pressure is easily accounted for and the occurrence of different nonclassical solutions is shown here to depend only on the reservoir conditions and the ambient pressure. The classification scheme of Cramer and Fry [3] is further refined by introducing suitable limit solutions which separate families of solutions that are qualitatively similar.

References

[1] M. S. Cramer and A. Kluwick, "On the propagation of waves exhibiting both positive and negative nonlinearity,", *J. Fluid Mech.* **142**, 9–37, 1984
[2] A. Kluwick, "Internal flows of dense gases,", *Acta Mech.* **169**, 123–143, 2004
[3] M. S. Cramer and R. N. Fry, "Nozzle flows of dense gases,", *Phys. Fluids A* **5**, 1246–1259, 1993

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