FORCED RESPONSE OF OSCILLATORS WITH DELAY AND HYSTERESIS

TAMAS KALMAR-NAGY

Department of Aerospace Engineering
Texas A&M University
College Station, TX, 77845, USA
usnctam@kalmarnagy.com

The dynamics of machine tool chatter in material removal processes is often modeled by delay-differential equations [5]. The nonlinear dependence of the cutting force on chip thickness introduces a variety of nonlinear phenomena. Most of the theoretical analyses of machine tool vibrations employ force laws that are based on the assumption of quasi-steady state cutting. However, cutting is a dynamic process and experimental results show that the cutting force-chip thickness relation exhibits hysteresis [1, 4]. This may be due to Coulomb friction or elastoplastic behavior of the material. Motivated by this problem we study the harmonically excited undamped oscillators with delay and hysteretic restoring force

$$\ddot{x} + x = pF\left(x\left(t-\tau\right),\varepsilon\right) + A\cos\left(\omega t\right)$$
$$\ddot{x} + x = pF\left(x\left(t-\tau\right) - x\left(t\right),\varepsilon\right) + A\cos\left(\omega t\right),$$

where $F\left(x,\varepsilon\right)$ is the hysteretic restoring force [2]. In the limit $\varepsilon \to 0$ the hysteretic loop vanishes while $\varepsilon = 1$ corresponds to elastic-perfectly plastic behavior. The forced response of these oscillators is characterized and nonclassical response [3] is also discussed.

[1] P. Albrecht, "Dynamics of the metal-cutting process", *Journal of Engineering for Industry*, 87:429–441, 1965.
[2] T. K. Caughey, "Sinusoidal excitation of a system with bilinear hysteresis", *Journal of Applied Mechanics*, 27:640–643, 1960.
[3] W. Lacarbonara and F. Vestroni, "Nonclassical responses of oscillators with hysteresis", *Nonlinear Dynamics*, 32:235–258, 2003.
[4] R. J. Szakovits and A. F. D'Souza, "Metal cutting dynamics with reference to primary chatter", *Transactions of the ASME*, B98(1):258–264, 1976.
[5] S. A. Tobias, *Machine Tool Vibration*, Blackie and Son Limited, London, 1965.

Keywords: metal cutting, delay-differential equation, hysteresis