MULTIPLE SCATTERING BY MULTIPLE SPHERES

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Suppose that we are interested in the scattering of sound by many small scatterers; for example, we might be interested in using ultrasound to determine the quality of certain composites, fresh mortar, or food products such as mayonnaise. If we knew the shape, size, and location of every scatterer, we could solve the multiple-scattering problem by solving a boundary integral equation, for example. However, usually we do not have this information. Thus, it is common to regard the volume containing the scatterers as a random medium, with certain average (homogenized) properties. Here, we are concerned with finding an *effective wavenumber*, K, that can be used for modelling wave propagation through the scattering volume. This is a classical topic, with a large literature: there are well-known papers by Foldy, Lax, Waterman and Truell, Twersky, and Fikioris and Waterman; for references and much discussion, see [1].

Let n_0 be the number of scatterers per unit volume. Assuming that n_0 is small, typical estimates for K are of the form

$$K^2 = k^2 + \delta_1 n_0 + \delta_2 n_0^2,$$

where $k = \omega/c$, the time-dependence is $e^{-i\omega t}$, c is the sound speed, and the coefficients δ_1 and δ_2 are to be found. The established formula for δ_1 is usually attributed to Foldy. However, there are many different formulas in the literature for δ_2.

In 1967, a formula for δ_2 was obtained by Lloyd and Berry [2]. Their formula is much more complicated than several others (it involves a certain integral of the far-field pattern for a single scatterer), and it was derived using methods and language coming from nuclear physics. Thus, in their approach, which they "call the 'resummation method', a point source of waves is considered to be situated in an infinite medium. The scattering series is then written out completely, giving what Lax has called the 'expanded' representation. In this expanded representation the ensemble average may be taken exactly [but then] the coherent wave does not exist; the series must be resummed in order to obtain any result at all."

We show that a proper analysis of the scattering problem leads to the Lloyd–Berry formula. Our analysis does not involve "resumming" series or divergent integrals. The details of the (three-dimensional) calculation are in [3]; a similar (but simpler) calculation in two dimensions is in [4].

References

[1] P.A. Martin, *Multiple Scattering: the Interaction of Time-Harmonic Waves with N Obstacles*, Cambridge University Press, 2006.

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[4] C.M. Linton and P.A. Martin, "Multiple scattering by random configurations of circular cylinders: second-order corrections for the effective wavenumber," *J. Acoust. Soc. Amer.* **117**, 3413–3423, 2005.

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