FLEXURAL VIBRATION AND DAMPING OF THERMOELASTIC THIN PLATES

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Recent interest in thermoelasticity of vibrating structures is motivated by the unavoidable fact that thermoelastic damping remains after all other sources of dissipation are removed, e.g. mounting losses, or crystal imperfection. Thus, carefully controlled experiments on silicon MEMS oscillators [1] have demonstrated that the dominant loss mechanism is thermoelastic (TE). Further reduction in TE damping of MEMS resonators, with applications to next generation RF devices, requires understanding this limiting effect. The talk reviews recent theoretical developments starting from a general procedure for calculating thermoelastic damping in structures [2]. The main idea is to take advantage of the fact that coupling between elastic and thermal fields is weak, and consequently TE damping can be found the purely elastic lossless solution by regular perturbation.

The talk emphasizes new results for TE damping in flexural vibration of thin structures. The procedure generalizes the original Zener [3] theory for beams, though for thin plate situation is more interesting by virtue of the two dimensional nature which introduces the possibility of flexure in two directions, coupling the kinematics and the thermal diffusion. The latter is due to the alternating compression and extension on opposite faces of the thin structure, causing the instantaneous or isentropic bending stiffness of the plate to relax to its isothermal value. It can be shown that the lateral (in-plane) diffusion of heat is negligible. The net effect for thin plates in flexure is to modify the equation relating bending moment $\mathbf{M}$ and curvature $\boldsymbol{\kappa}$ to

$$\mathbf{M} = \mathbf{D}^{(0)}\boldsymbol{\kappa} - \Delta\mathbf{D}g * \boldsymbol{\kappa},$$

where $\mathbf{D}^{(0)}$ is the instantaneous or adiabatic bending stiffness, $\Delta\mathbf{D} = \frac{T}{IC_p}\mathbf{D}^{(0)}\boldsymbol{\alpha}\boldsymbol{\alpha}^t\mathbf{D}^{(0)}$ and $\boldsymbol{\alpha}$ is the thermal expansion. The scalar relaxation function $g(t)$ is the central property governing TE damping in thin plates and it will be discussed in detail, with comparisons to prior theories, e.g. Chadwick's exact but overly complicated model [4], Simmond's correct but simplified equation [5], and others [6], but none as concise as the above equation indicates. In short, the relaxation function replaces the coupled equations of thermoelasticity and flexural vibration by an equivalent *viscoelastic* thin plate theory. The boundary conditions associated with the viscoelastic model are important. For example, the damping of a plate with fixed edges is found by simply replacing the wave number with the complex-valued one that follows from the above moment equation. If not all edges are clamped, then the boundary conditions play an important role. For instance, the limiting case of Zener's beam is recovered but only with the proper edge conditions and the resultant implications for damping. Refinements to the classical TE model will be discussed, including quantum mechanical limitations on phonon effects in very thin plates.

References

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