ANALYTICAL MODELS OF VORTEX WAKES

MARK A. STREMLER

Department of Mechanical Engineering
Vanderbilt University
Nashville, Tennessee 37235, USA
mark.stremler@vanderbilt.edu

Th. von Kármán was the first to consider a singly periodic array of point vortices as a model of vortical flow in the laminar wake of a bluff body [1]. His analysis consisted of two infinite rows of oppositely signed vortices. This simplified analytical approach, which remains the *only* analytical approach to modeling vortex wakes, provided a means for investigating wake stability and for estimating wake-induced drag on the body. Despite the utility of this model, research using singly periodic point vortex arrays to study laminar wakes has tended to focus quite specifically on the staggered vortex street. I will present the use of point vortex models for investigating equilibrium configurations and streamline topology in oblique and exotic wakes.

When a wake is modeled as two oppositely signed vortices in a singly periodic strip, any choice of vortex positions gives a uniformly translating relative equilibrium. In the case of the staggered Kármán street, the wake translates along its length, and adjacent co-moving points (i.e., stagnation points in a frame moving with the vortices) are joined by streamlines. For small deviations from this ideal case, however, the wake translates obliquely, and the streamline structure becomes more intricate. Fluid entrained in the wake can be wrapped around many of the vortices before passing through to the other side. I will discuss the bifurcations that occur in the streamline topology of obliquely translating vortex streets and the influence of this structure on mixing in wakes.

The Kármán vortex street appears consistently in the wakes of rigid bluff bodies. If the body is instead free (or forced) to oscillate, the resulting wake structure can be much more exotic, with two, three, four, or more vortices shed per period [2]. I will present relative equilibrium configurations of singly periodic point vortex systems as models of these exotic wakes. When three vortices are shed per period, the possible configurations can be determined systematically [3]. The focus here will be on those configurations that translate along the wake. The streamline topology in various exotic wake models will also be discussed.

References

[1] Th. von Kármán, "Über den Mechanismus des Widerstandes, den ein bewegter Körper in einer Flüssigkeit erfährt. 1. Teil.," Göttinger Nachrichten Math-Phys. Klasse **1911**, 509–517, 1911; Th. von Kármán, "Über den Mechanismus des Widerstandes, den ein bewegter Körper in einer Flüssigkeit erfährt. 2. Teil.," Göttinger Nachrichten Math-Phys. Klasse **1912**, 547–556, 1912.
[2] C.H.K. Williamson and A. Roshko, "Vortex formation in the wake of an oscillating cylinder," J. Fluids Struct. **2**, 355–381, 1988.
[3] M.A. Stremler, "Relative equilibria of singly periodic point vortex arrays," Phys. Fluids **15**(12), 3767–3775, 2003.

Keywords: vortices, laminar wakes