

## TWO-FLUID DROP BREAKUP

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The break-up of a liquid thread or drop under the influence of capillary forces has been the subject of numerous theoretical and experimental studies over the past decade, motivated both by industrial applications (e.g. spraying, emulsification, ink-jet printing) and by theoretical interest in topological transitions. Capillary break-up and the subsequent recoil are examples of finite-time singularities in which the interfacial curvature becomes infinite at the point of disconnection. The divergence of scales near disconnection allows similarity solutions to be found for the flow close to disconnection for both inviscid (potential) and very viscous (Stokes) flow [1–9]. Despite substantial progress, a number of outstanding questions remain unanswered. This paper discusses some of the relationships between the known solutions and these outstanding questions.

The inviscid problem is characterised by the density ratio  $\Delta$  of internal to external fluids and the viscous problem by the corresponding viscosity ratio  $\lambda$ . For nonzero  $\Delta$  the similarity lengthscales are all  $\tau^{2/3}$  and for finite nonzero  $\lambda$  they are all  $\tau$ , leading to well-understood self-similar behaviour. The cases  $\Delta = \infty$ ,  $\lambda = 0$  and  $\lambda = \infty$  are all special and correspond to singular limits. Time-dependent simulations for large  $\Delta$  show overturning instabilities due to a Kelvin–Helmholtz-like mechanism, and the case  $\Delta = \infty$  is the subject of some controversy! We present our latest results.

### References

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