NUMERICAL MODELLING FOR THE RESONANT BEHAVIOR OF ELASTIC PLATES LOADED WITH GRANULAR LAYERS

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Understanding the resonant behavior of a granular layer on top of an elastic plate opens the possibility of designing more reliable land-mine detection via acoustic/seismic methods. Experimentally, it has been observed (see [1] and references therein) that the resonant frequencies of a elastic-granular system, composed of an elastic plate clamped on its boundary with a layer of granular material on top of it, are strongly dependent on the particle size. The general trend of the resonant frequency for this system as a function of the granular layer mass (or thickness) reaches a minimum and then increases due to an effective bending stiffness of the granular layer.

One-dimensional non-linear models (see [2]) cannot explain the variation of the effective bending stiffness behavior with the particle size. We first propose to use the finite element method for discretizing the plate and the equivalent elastic layer with which we replace the actual granular layer. To determine the effective elastic parameters of the equivalent elastic layer, one could employ the methods developed in [3], [4]. As discussed in [5], however, the effective moduli obtained from these theories differ from the experimental and explicit numerical simulations of granular materials by 70% to 150% (for the shear modulus).

We propose an alternative computational-experimental method for finding the effective elastic moduli of the equivalent elastic layer (which has the same density and thickness as the granular layer). We use the experimental resonant frequencies and apply an inverse method to recover the elastic moduli. A nonlinear shooting method is used between iterations of the FEM solution until convergence to a tolerance of 1% is achieved. The effective Young's moduli obtained show the increase in stiffness with the increase of the particle size. Not surprisingly, for small thicknesses of the granular layer the inverse problem is highly ill-posed in the sense that small perturbations of the input resonant frequencies can result in large differences in the recovered effective Young's modulus values.

As the "effective continuum"-based approach is not predictive (it needs the experimental resonant frequency data), we develop a coupled model that combines the Discrete Element Method (for the granular material) and the Finite Element Method for the vibrating plate. We analyze the resonant behavior of a reduced number of particles (hundreds) and discuss plans for modelling more realistic numbers.

References

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