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GBT FORMULATION TO ANALYZE THE BUCKLING BEHAVIOR OF THIN-WALLED MEMBERS SUBJECTED TO NORMAL STRESS GRADIENTS AND SHEAR STRESSES

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ABSTRACT

The structural efficiency of a given steel member can only be adequately assessed after acquiring in-depth information concerning its buckling behavior, a task that involves (i) the identification of the relevant buckling modes and (ii) the evaluation of the associated bifurcation stresses. In practice, the vast majority of thin-walled steel members (namely beams or beam-columns) are subjected to normal stress distributions varying along their lengths (*e.g.*, the stress distributions due to distributed transverse loads or non-uniform torsion), which automatically entail the presence of pre-buckling shear stresses. However, it seems fair to say that practically all of the available information about the influence of the pre-buckling normal stress gradients and shear stresses on the member buckling behavior deals with the lateral-torsional buckling of beams. As far as the local-plate or distortional buckling behaviors are concerned, only a fairly small number of quite recent studies dealt with members acted by longitudinally non-uniform applied stress distributions (mostly beams, *i.e.*, members under pure bending). Among the studies on stress gradient effects, it is worth mentioning the experimental work due to Yu (2005), the shell finite element analyses reported by Yu & Schafer (2004) and Yu (2005) and the finite strip investigation carried out by Chu *et al.* (2005) – however, it should be pointed out that the finite strip model developed by these authors did not incorporate pre-buckling shear stresses, an omission that may affect considerably the accuracy of the results obtained.

In the last decade, Generalized Beam Theory (GBT – Schardt 1989-1994) emerged as a very competitive method to analyze the geometrically non-linear behavior of prismatic thin-walled members – it was extensively employed by Davies and his co-workers to perform buckling analyses of cold-formed steel members (Davies 1994-2000). GBT is a beam theory that (i) accounts for both cross-section and member deformations, (ii) approximates the cross-section deformed configuration by means of a linear combination of structurally meaningful "deformation modes" and, due to its unique modal features, (iii) provides a general, clarifying and elegant approach to obtain accurate solutions for several non-linear problems involving thin-walled members. However, up until now all the buckling analyses were carried out exclusively in the context of members subjected to uniform compression and/or bending – *i.e.*, members without either normal stress gradients or shear stresses. Since no fundamental limitation exists, it seems fair to say that this fact stemmed from the lack of a GBT formulation and numerical implementation that properly incorporates these two aspects.

The aim of this paper is (i) to present the main steps involved in the development and numerical implementation (beam finite element) of a GBT formulation able to handle normal stress gradients and shear stresses and intended to analyze the local and global buckling behavior of thin-walled members subjected to non-uniform internal force and moment diagrams, and also (ii) to illustrate its application. First, one derives the system of GBT equilibrium equations that take into account (i) the variation, along the member length, of the axial force, major and minor axis bending moments and bi-moment, as well as (ii) the presence of the corresponding shear stresses. Then, adopting Hermite cubic polynomials to approximate the displacement field, the linear and geometric stiffness matrices are established – the latter is a linear combination of four matrices, associated with uniform, linear, quadratic or cubic applied stresses. Finally, one employs the developed GBT formulation/implementation to analyze the local-plate, distortional and global buckling behavior of lipped channel and I-section beams acted by non-uniform bending moments – for validation purposes, some GBT-based critical stresses are compared with values either reported in the literature (global buckling) or yielded by shell finite element analyses (local-plate and distortional buckling).