NONLINEAR MAGNETOELASTIC MATERIALS

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Commercial applications of magneto-sensitive (MS) elastomers have recently been developed in the form of, for example, controllable membranes, controllable stiffness devices and rapid response interfaces designed to optimize mechanical systems. MS elastomers rapidly and significantly change their stiffness on the application of a magnetic field and are composed, in general, of an elastomeric matrix with a volumetric distribution of micron-sized ferrous particles added during the vulcanization process.

There is an increased demand for suitable constitutive equations for the analysis and solution of representative boundary-value problems. Several alternative formulations of constitutive equations have recently been proposed to describe the highly nonlinear magnetoelastic interactions. They are based on a modified free energy function that depends, in addition to the deformation gradient tensor, on the magnetic induction vector as the independent magnetic variable (alternatively the magnetic field vector). Selected publications are [1, 2, 3, 4] and references therein.

In this presentation we summarize the magnetic and mechanical balance equations and boundary conditions. Then, the constitutive equations for the magnetoelastic interactions are given for compressible and incompressible magnetoelastic materials following the derivations in [1]. These equations are specialized to incompressible and isotropic magnetoelastic materials using separately the magnetic induction and the magnetic field vector as an independent variable.

Representative boundary value problems are solved to highlight the increase in material stiffness associated with an increase in the magnetic field. We first consider an infinite slab of incompressible isotropic magneto-sensitive material with an applied magnetic field normal to the top and bottom faces. The slab is then subject to simple shear. In the second example we consider the extension and inflation of a circular cylindrical tube of finite length. The finite dimension of the tube raises difficulties with the magnetic compatibility conditions and therefore no simple analytical solution is attainable. A numerical solution procedure is used and reduced to the determination of two scalar potentials, one for inside the body and the second for the surrounding vacuum, with appropriate continuity conditions on the bounding interface. The distribution of the magnetic field inside the deformed body, for different aspect ratios of the tube, is determined as a function of the applied deformation.

References

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