CONTINUUM MODEL OF A NEWTONIAN SYSTEM OF INTERACTING PARTICLES FOLLOWING A LIE GROUP FLOW

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The theories of continuum and Newtonian particle systems are well axiomatized mechanical-mathematical theories. The theory of the mathematical continuum was introduced (among many reasons) to describe the real particle systems present in the universe. Although the continuum seemed to be the right tool, many of the particle systems can not be described by the continuum sufficiently to call it a model of the latter (i.e. the particles). It is to be noted that the particles referred to are Newtonian and not (sub)nuclear ones for which a Lagrangian would have to introduced to determine the mass, and then the force and acceleration would be related through the Newtonian equation F=ma, where the force is derived from a multi-particle potential.

The continuum and particle models are introduced through two categories: Con and Par, and their interaction through a functor F: Con $\rightarrow$ Par, see [1]. The structure (Con, Par, F) is regarded as a model of the particle system Par, if the Par system can be imbedded in the Con system, i.e. by identifying certain particles of B, being topological point sets, with those of Par. The existence of F will be attempted on the basis of morphisms and such objects as forces. It will be demonstrated that it is possible to develop a kinematical model, but a dynamical model for a long-range interaction among the elements of the Par system may not exist at all. The dynamical model is based on a norm in the space of admissible displacements in the Con system, constructed as Whitney extensions of the Par system, and Gateau differential in the infinite-dimensional set of model forces determined through a dynamical process (t , b) , which is an element of a Cartesian product (TxB), where T is a set of stress tensors and B is a set of body forces. The model forces will be regarded as a superset of a typical Embedded Atom Method.

The motion of particles in the set Par will be carried out through a one-parameter Lie group, where the particles will be subjected to translations and rotations. Therefore, equivalence classes of Con-displacements in the continuum manifolds will be introduced, to simplify the concepts [see 2-3]. An example of a successful continuum model will be provided.

[1] H. Zorski, “Non-existence of a Continuum that Models a Newtonian System of Interacting Particles”, Arch. for Rat. Mech. And anal., Vol.56, No.4, 1974, pp. 320-333.
[2] J.Roj,“On Some Equivalence Classes of Configurations and Deformations of a Continuum”, Bulletin De L’Academie Polonaise Des Sciences, Vol XXVIII, No.7-8, pp.31-34, 1980.
[3] J.Roj,“On Some Global One-parameter Lie Subgroups of a Group of Deformations of a Continuum”, Bulletin De L’Academie Polonaise Des Sciences, Vol XXVIII, No.7-8, pp.35-39, 1980.

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