# POSTBUCKLING AND DYNAMICS OF A CLAMPED HEAVY BEAM

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A beam is called "heavy" if its self-weight is included in the analysis. An ideal, slender, straight beam resting on a flat, rigid foundation does not buckle when subjected to a finite compressive load, since the load cannot overcome the effect of the beam's weight. However, it may buckle due to imperfections or if its ends are moved toward each other.

Upheaval buckling of a heavy beam (actually a beam-column) resting on a rigid horizontal foundation has been investigated for various situations, including pipelines resting on the seabed, handling of fabric or paper, railroad tracks, and rock strata. Papers on these problems include [1,2] and earlier publications cited therein. Most of these studies consider equilibrium configurations and perform a linear analysis. Often end conditions are not involved, and local buckling occurs in an internal segment of the beam. An inclined foundation was considered by Bogy and Paslay [3], in relation to buckling of a drill pipe in an inclined hole. The bottom end of the heavy beam was simply supported, and the beam was subjected to a compressive load.

Here the heavy beam (or strip) is treated as an inextensible elastica. Its ends are fixed (no transverse displacement or rotation), and one end is pushed toward the other. Postbuckling of such a beam is examined, both theoretically and experimentally, for horizontal and inclined foundations. A nonlinear analysis is performed to compute equilibrium shapes for large deflections, including cases in which self-contact occurs. Both horizontal and inclined (including vertical) foundations are considered. In addition, the case of no foundation is considered, where there is no restriction on the direction of beam deflection. For this case, stable and some unstable solutions are found. If the beam is buckled upward and then the ends are pulled apart, snap-through occurs.

In addition to the static analysis, small vibrations about the buckled equilibrium states are investigated. A shooting method is applied to solve the governing equations numerically. In addition, experiments are carried out with polycarbonate strips. The correlations between the experimental and analytical equilibrium shapes, vibration modes, and vibration frequencies are very good.

Finally, large motions about a postbuckled equilibrium configuration are analyzed. A Galerkin-type method is applied to the dynamic elastica equations, using shape functions that correspond to the small-vibration mode shapes. The resulting nonlinear differential equations for the time-varying amplitudes of the geometric and internal-force variables are integrated numerically, and time histories corresponding to varying initial conditions are obtained.

### References

[1] J.G.A. Croll, "A simplified model of upheaval thermal buckling of subsea pipelines," *Thin-Walled Struc.* **29**, 59–78, 1997.

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[3] D.B. Bogy and P.R. Paslay, "Buckling of drill pipe in an inclined hole," *ASME Engrg. Indus.*, **86**, 214–220, 1964.

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