Stability of mechanical systems with negative stiffness components

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Potential applications for high-damping and high-stiffness composites [1] motivate extensive research on the effects of negative-stiffness inclusions to the overall properties of composites. In structural mechanics, negative stiffness is obtainable through post-buckling processes. In solid materials, the Landau phenomenological theory of phase transformations predicts negative curvature in the system free-energy, i.e. a negative stiffness in the vicinity of phase transitions.

Recent theoretical advances have been carried out based on the Hashin-Shtrikman composite models [2] and one-dimensional discrete viscoelastic systems [3]. Here, we analyze a two dimensional triangular structure that contains pre-selected negative-stiffness components, to study its underlying deformation mechanisms and stability. The stiffness and damping anomalies are identified both in the bulk and shear deformation modes. Furthermore, we study structure-deformation evolution with respect to the magnitude of negative stiffness under bulk or shear loading, and the phenomena related to dissipation-induced destabilization and inertia-induced stabilization, according to the Lyapunov stability analysis. The evolution shows strong correlations between stiffness anomalies and deformation modes. Our stability results reveal that stable damping peaks, i.e. stably extreme effective damping properties, are achievable under hydrostatic loading when the inertia is greater than a critical value. Moreover, destabilization induced by elemental damping is observed with the critical inertia. Regardless of elemental damping, when the inertia is less than the critical value, a weaker system instability is identified.

References

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