# Modal Data for a Piezoelectric Cylinder and Their Applications

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## Abstract

The spectral decomposition of the governing differential operator for layered piezoelectric circular cylinder yields a complete set of modal data. These data consist of all propagating modes as well as edge vibrations for the cylinder. In this seminar, we show the construction of steady-state Green's functions and explore the reflection of monochromatic waves arriving at the free end of the cylinder using these modal data

The structure is in the form of a right circular cylinder composed of any number of perfectly bonded layers, each of constant thickness and distinct piezoelectric properties. The cylinder may be hollow or solid, whose lateral surfaces are traction-free. The equations of motion are based on linear piezoelectricity and semi-analytical finite element modeling. Upon inserting a wave-like solution form into the equations of motion, the following Hermitian system of equations is obtained.

$$\Big[\big(\mathbf{K_1} + m^2\mathbf{K_4} + mk_m\mathbf{K_5} + k_m^2\mathbf{K_6} - \omega^2\mathbf{M}\big) + i\big(m\mathbf{K_2} + k_m\mathbf{K_3}\big)\Big]\mathbf{V_m} = 0$$

The circumferential mode number $m$ is always assigned, leaving a two parameter eigenproblem where either axial wave number $k_m$ or squared circular frequency $\omega^2$ can act as the eigenvalue parameter. Both eigenproblems give data in useful forms for formulation of Green's functions and end reflection phenomena.

For incoming axisymmetric waves, there is a particular frequency which enables an end resonance condition, a phenomenon characterized by extremely high amplitudes of end motions at a certain frequency *vis-a-vis* those of neighboring (i.e., slightly different) frequencies.

In addition to passive reflections, it is possible to impose some voltage distribution over the free end to alter (or control) the reflection. In fact, with an out-of-phase applied voltage, it is possible to extract electrical energy from it, i.e., energy harvesting.