Invariant Configurational Integrals for Geometrically Necessary Dislocations.

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Abstract

Recently, in a unpublished work, we have derived and discovered several classes of configurational invariant integrals that represent the symmetry and the general characterizations of the compatibility or incompatibility conditions of a continuum. Some of the integrals have clear physical meanings as they are the manifestation or the density of geometrically necessary dislocation (GND).

We speculate that these integrals are intimately related to the cohesive strain-gradient elasticity or the strain-gradient plasticity theories.

Unlike Eshelby's energy-momentum tensor or Rice's J-integral, these invariant configurational integrals are derived purely from geometrical or topological conditions, bearing no connections to materials' constitutive relations nor thermodynamics theories.

In the preliminary study, we have found that these integrals are invariant or path-independent when the density of the GND inside the integration contour is zero.

For instance, we discover that the following tensor,

$$S_{ij} = \frac{1}{2}\alpha_{mn}\kappa_{mn}\delta_{ij} - \theta_{k,i}\alpha_{jk}$$

termed as the incompatibility-momentum tensor, is invariant (path-independent) in a perfect plasticity medium, if GND density is zero inside the contour of the following integral,

$$L_i = \oint_{\partial\Omega} S_{ij} n_j dS = 0.$$

where α is the GND density tensor, κ is the curvature tensor, and θ is the rotation vector. The presentation is mainly centered at our interpretations of meanings of these invariant integrals, because we believe that a correct interpretation of these integrals may lead a breakthrough in dislocation mechanics and configurational force mechanics.