ON A MODIFIED COUPLE STRESS THEORY

X.-L. Gao* and **S. Park**
Department of Mechanical Engineering, Texas A&M University,
3123 TAMU, College Station, TX 77843-3123
* E-mail: xlgao@tamu.edu

Lacking an internal length scale, classical elasticity and plasticity cannot be used to interpret the size effect observed in numerous tests at micron and nanometer scales. Higher-order (nonlocal) continuum theories contain additional material parameters and have been proven to be capable of effectively explaining various size effects. Couple stress theories represent one class of such higher-order theories, to which Mindlin made significant contributions (e.g., [1, 2]).

Recently, a modified couple stress elasticity theory was proposed by Yang et al. [3]. Compared to the classical couple stress theory, the modified theory has two advantages: the couple stress tensor is symmetric, and only one internal length scale parameter is involved. These features make the modified theory easier to use. A new model for the bending of a Bernoulli-Euler beam has been developed [4] using this modified couple stress theory.

In the current communication, a variational formulation based on the total potential energy principle is provided for the modified couple stress theory mentioned above, with all governing equations and boundary conditions explicitly derived in one process. Both the stress and displacement methods are utilized in the formulation. As a direct application of the presently obtained displacement form of the modified couple stress theory, a simple shear problem is analytically solved. Sample numerical results are also presented to illustrate the new solution, which contains an internal material length scale, and to compare it with the classical elasticity based solution.

References

[1] Mindlin, R. D. and Tiersten, H. F., 1962, Effects of couple-stresses in linear elasticity. *Arch. Ration. Mech. Anal.* **11**, 415-448.

[2] Mindlin, R. D., 1964, Micro-structure in linear elasticity. *Arch. Ration. Mech. Anal.* **16**, 5-448.

[3] Yang, F., Chong, A.C.M., Lam, D.C.C. and Tong, P., 2002, Couple stress based strain gradient theory for elasticity, *Int. J. Solids Struct.* **39**, 2731-2743.

[4] Park, S. K. and Gao, X.-L., 2006, A new Bernoulli-Euler beam model based on a modified couple stress theory. To appear in *Proc. of the 10th ASCE Aerospace Division International Conference on Engineering, Construction and Operations in Challenging Environments*, League City/Houston, TX, March 5-8, 2006.

Keywords: Couple Stress, Simple Shear, Size Effect, Potential Energy