

# HIGH FREQUENCY VIBRATIONS OF HYGROTHERMOPIEZOELECTRIC THIN SHELLS

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This paper deals with the high frequency time harmonic free vibrations of a hygrothermopiezoelectric functionally graded thin shell of uniform thickness. The essential ingredients of the paper is to introduce the couple effects, functionally graded material and invariant fully variational form into the 2-D equations of a thin shell (cf., [1]). In the first part of the paper, a moisture field vector and a temperature field vector are introduced as the gradient of moisture concentration and temperature increment, respectively. Hence, the consistency of the fundamental equations of coupled mechanical, electrical, thermal and moisture fields is maintained [2]. Then, the fundamental equations are recorded in differential form. Alternatively, the fundamental equations are expressed in variational form through a unified variational principle deduced from a general principle of physics (e.g., the principle of virtual work) by modifying it using an involutory (or Legendre's) transformation. The variational principle leads to the fundamental equations as its Euler-Lagrange equations (cf., [3]) where the mechanical, thermal and moisture fields are considered).

In the second part, the geometry of the shell is described, some preliminaries for the surface geometry is recorded, and the displacement, moisture and thermal field vectors and the electric potential are represented by the power series expansions in the thickness coordinate of the shell, as a generalisation of Love's first and second approximation of elastic shells. The elastic material of the shell is taken to be functionally graded across the thickness and the constitutive coefficients and the mass density of the shell are expressed by the power series expansions in the thickness coordinate as well. The variational principle together with the series expansions is used to consistently deduce a system of 2-D equations of the shell from the 3-D fundamental equations in both invariant differential and fully variational forms. The governing equations consist of the 2-D divergence equations, gradient equations and constitutive relations, and the boundary and initial conditions to supplement them. The 2-D governing equations of shell may be readily expressed in any particular system of coordinates most suitable to the geometrical configuration at hand due to their invariant form. The governing equations account for the extensional, thickness shear, flexural and coupled vibrations of the shell.

In the third part, certain cases involving special geometry, types of vibration and material properties are indicated. The cases are reported by omitting some of the effects, the curvature and the functionally grading of materials. Besides, the uniqueness is investigated and the conditions sufficient for the uniqueness are enumerated.

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## References

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