COMPLETELY REGULARIZED INTEGRAL EQUATIONS FOR THREE-DIMENSIONAL, SECOND ORDER, ELLIPTIC PROBLEMS

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A systematic technique has been adopted to establish a weakly-singular, weak-form state variable integral equation and a weakly-singular, weak-form flux integral equation for three-dimensional, second order, elliptic boundary value problem. The crucial features of this pair of integral equations are that they contain only weakly-singular kernels of order $1/r$, and that their validity requires only continuity of the state variable on the boundary. The development has been carried out within a general context with final results applying to several physical problems, e.g. Darcy's flow in porous media, steady-state heat conduction problems, elasticity problems, problems involving fractures in either elastic media or piezoelectric media.

The key step in the regularization procedure (analogous to that employed by [1,2]) is to construct special representations for both the flux fundamental solution and the strongly-singular kernel which is present in the flux integral equation. These representations are in a form which is well-suited for an integration by parts to be performed, with applications of Stokes' theorem, on the standard integral relations. The weakly-singular kernels appearing in these representations have been constructed by the method of an integral transform with final results being in terms of a unit contour integral.

A proper use of the weakly-singular, weak-form integral equations for the state variable and the flux gives rise to a symmetric formulation governing the boundary value problem. This formulation can be used as a basis for the development of a numerical procedure based upon a weakly-singular, symmetric Galerkin boundary element method (SGBEM). Since the formulation is weakly-singular in nature, standard continuous elements can be employed everywhere in the discretization of the boundary.

References

[1] J. Rungamornrat and M.E. Mear, "Weakly-singular, weak-from integral equations for cracks in three-dimensional anisotropic media," Technical report: ICES report 05-24, The University of Texas at Austin, 2005.
[2] J. Rungamornrat and M.F. Wheeler, "Weakly-singular integral equations for Darcy's flow in anisotropic porous media," *Engineering Analysis with Boundary Elements*, Article in press, 2006.

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